Conclusion Notes

**Paragraph 1: What we did and why we did it**

We proposed two methods for decreasing the computational cost of constructing Jacobian matrices for analytic inversions of satellite observations of atmospheric constituents and demonstrated these methods in an inversion of GOSAT column methane observations for July 2009 over the North American domain. We found that both methods are capable of substantially reducing the computational cost of constructing the Jacobian matrix while accurately constraining posterior emissions where information content is highest. In an era where satellites provide increasingly high-resolution, dense observations of atmospheric constituents, analytic inversions of linear systems can improve constraints on emission sources and characterize the associated errors and information content. Previously, the analytic approach was limited by the computational cost of constructing the Jacobian matrix, which required a forward model simulation for every state vector element constrained by the inversion. Our methods allow analytic inversion of satellite observations at high-resolution with many fewer forward model simulations.

**Paragraph 2: Analytic solution to the inverse problem and optimal reductions in dimension and rank**

We considered inverse problems where the forward model is linear with respect to the state vector elements. In this case, there exists an analytic solution for the minimum of a Bayesian cost function that is a function of the prior estimate and its errors, the observations and their errors, and the forward model, as expressed by the Jacobian matrix. The analytic solution yields closed-form characterization of the posterior emissions, their errors, and their information content. In systems with a known Jacobian matrix, optimal reductions in the dimension or rank of the inversion maximize the information content. In systems without a known Jacobian matrix, an initial estimate of the Jacobian matrix can be constructed at no computational cost using a mass balance approach. This initial estimate together with the prior and observational errors can accurately quantify the fine structure of information content in the inversion. We use this initial estimate of information content to define optimal reductions in the dimension and rank of the state vector, decreasing the computational cost of constructing the Jacobian matrix.

**Paragraph 3: Reduced dimension construction**

Our dimension reduction method uses the initial estimate of the averaging kernel

**Paragraph 4: Reduced rank construction**

Our rank reduction method uses the initial estimate of the averaging kernel matrix

**Paragraph 5: Results summary (reduced dimension)**

**Paragraph 6: Results summary (reduced rank)**

**Paragraph 7: Next steps**